

Collins effect in semi-inclusive deep inelastic scattering process with a ${}^3\text{He}$ target

Jun She

*School of Physics and State Key Laboratory of Nuclear Physics and Technology,
Peking University, Beijing 100871, China*

Bo-Qiang Ma*

*School of Physics and State Key Laboratory of Nuclear Physics and Technology,
Peking University, Beijing 100871, China and
Center for High Energy Physics, Peking University, Beijing 100871, China*

Abstract

We re-examine our previous calculation on the Collins effect in semi-inclusive deeply inelastic scattering (SIDIS) process with a ${}^3\text{He}$ target, and find that our previous treatment on the dilution factors may cause the results larger than the realistic situation. We thus modify our calculation in an improved treatment with an updated prediction on the $\sin(\phi_h + \phi_S)$ asymmetry for the JLab 12 GeV under the transverse momentum dependent (TMD) factorization framework. Meanwhile, we also provide the prediction of such asymmetry for the JLab 6 GeV and the prediction of the $\sin(3\phi_h - \phi_S)$ asymmetry related to pretzelosity.

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*Electronic address: mabq@pku.edu.cn

I. INTRODUCTION

The transverse spin structure of the nucleon is a hot issue in spin physics. Single spin asymmetry (SSA) through semi-inclusive deeply inelastic scattering (SIDIS) process with a transversely polarized target provides us with a plausible way to explore the internal structure of the nucleon. In such process a number of form factors with different azimuthal angle dependence are measurable. These form factors can be interpreted as the convolution of the parton distribution functions and fragmentation functions. According to the transverse momentum dependent (TMD) factorization [1], which is intuitively a natural generalization of the collinear case by taking into account the transverse momentum, eight TMD distribution functions are needed to describe the spin structure of the nucleon at the leading twist. Among the eight, some of them relating to the transverse spin effect have attracted many interests in recent years, such as the Sivers, transversity and newly pretzelosity functions, which can manifest themselves through the $\sin(\phi_h - \phi_S)$, $\sin(\phi_h + \phi_S)$ and $\sin(3\phi_h - \phi_S)$ asymmetries, respectively in the SIDIS process.

The HERMES Collaboration has already published their data on the proton target [2] with non-vanishing $\sin(\phi_h - \phi_S)$ and $\sin(\phi_h + \phi_S)$ asymmetries. Some unexpected phenomena were also observed, for example, the $\sin(\phi_h + \phi_S)$ asymmetry for π^- production is larger than that for π^+ production. A possible explanation for this is that the corresponding unfavored fragmentation function is comparable with or even larger than the favored one [3], i.e., $|H_{1\text{unf}}^\perp| \gtrsim |H_{1\text{fav}}^\perp|$, where H_1^\perp is the so called Collins function [4]. This relation is now widely adopted in parametrizations, but is still a puzzling theoretical problem. The COMPASS Collaboration [5] has also made the measurement with the deuteron target, which can be viewed as a combination of a proton and a neutron, and observed nearly zero asymmetries for both $\sin(\phi_h - \phi_S)$ and $\sin(\phi_h + \phi_S)$ asymmetries. This could be accounted for as the cancellation between the u and d quarks. So a direct measurement with the neutron target is ideal to address this issue. However, there is no free neutron target for the experiment, so usually the ${}^3\text{He}$ target is used as a substitution. ${}^3\text{He}$ is a spin half nucleus, composed by two protons and one neutron, with the polarizations of the two protons anti-parallelled, thus ${}^3\text{He}$ can be viewed as a nearly free neutron. Jefferson Lab (JLab) is performing the measurement with a ${}^3\text{He}$ target under 6 GeV, and will upgrade their plan by a 12 GeV beam energy in the near future [6]. Such plan can be considered as performing a measurement for

the purpose to focus attention on the neutron target.

However, after taking into account other effects, ${}^3\text{He}$ cannot be viewed as a pure neutron strictly. How to connect the experiment result of the ${}^3\text{He}$ target with the free neutron case needs to be carefully considered. Some theoretical work [7] already investigated the problem, and the following relation is often used,

$$A_{{}^3\text{He}} = p_n \cdot f_n \cdot A_n + 2p_p \cdot f_p \cdot A_p, \quad (1)$$

where $p_{p(n)}$ are the effective nucleon polarizations, with $p_p \approx -0.028$ and $p_n \approx 0.86$ as suggested in Ref. [7]. $f_{p(n)}$ are the dilution factors with the definitions

$$f_{p(n)}(x, z, \mathbf{P}_{h\perp}) = \frac{\int \sum_q e_q^2 f_1^{q,p(n)}(x, \mathbf{p}_\perp) D_1^{q,h}(z, \mathbf{k}_\perp)}{\sum_{N=p,n} \int \sum_q e_q^2 f_1^{q,N}(x, \mathbf{p}_\perp) D_1^{q,h}(z, \mathbf{k}_\perp)}. \quad (2)$$

We have simply generalized the expression above into the TMD case, and the integrals in the numerator and denominator are performed over all the phase space independently. The validity of Eq. (1) was discussed in Refs. [7]. In our previous paper [8], we already used this formula to predict the ${}^3\text{He}$ asymmetry, but with a constant value estimation for the dilution factors, adopted $f_p \approx 0.34$ and $f_n \approx 0.32$ around $z \approx 0.5$ as proposed [9]. Now we have re-examined our calculation, and find that this rough estimation is not adequate to give the proper prediction. In this brief note, we deal with the problem in an improved treatment with the dilution factors varying with kinematic variables, meanwhile we update our prediction from the following aspects:

- We present predictions not only for JLab 12 GeV, but also for JLab 6 GeV to confront with the data that will come soon.
- We calculate under the TMD framework, and show the asymmetries depending on x , z and $\mathbf{P}_{h\perp}$.
- We multiply only the azimuthal angle dependence as the weighting function to extract the corresponding asymmetry, rather than multiply an extra momentum dependent factor as we did in our previous paper. The former treatment is often used in many experiments, while the latter one is not so widely adopted, although it can avoid the tangle of the quark internal momenta from a theoretical viewpoint.
- We also give the asymmetry related to pretzelosity, another chiral-odd but T-even TMD distribution, which also couples with the Collins functions in the SIDIS process.

II. NUMERICAL CALCULATION

We write down the cross-section with the Collins effect terms [10, 11]

$$\frac{d^6\sigma_{UT}}{dxdydzd\phi_S dz d^2\mathbf{P}_{h\perp}} = \frac{2\alpha^2}{sxy^2} \left\{ (1-y + \frac{1}{2}y^2) F_{UU} + S_\perp(1-y) \right. \\ \times [\sin(\phi_h + \phi_S) F_{UT}^{\sin(\phi_h + \phi_S)} + \sin(3\phi_h - \phi_S) F_{UT}^{\sin(3\phi_h - \phi_S)}] + \dots \right\}, \quad (3)$$

with

$$F_{UU} = \mathcal{F}[f_1 D_1], \quad (4)$$

$$F_{UT}^{\sin(\phi_h + \phi_S)} = \mathcal{F}\left[-\frac{\hat{\mathbf{h}} \cdot \mathbf{k}_\perp}{M_h} h_1 H_1^\perp\right], \quad (5)$$

$$F_{UT}^{\sin(3\phi_h - \phi_S)} = \mathcal{F}\left[\frac{2(\hat{\mathbf{h}} \cdot \mathbf{p}_\perp)(\mathbf{p}_\perp \cdot \mathbf{k}_\perp) + \mathbf{p}_\perp^2(\hat{\mathbf{h}} \cdot \mathbf{k}_\perp) - 4(\hat{\mathbf{h}} \cdot \mathbf{p}_\perp)^2(\hat{\mathbf{h}} \cdot \mathbf{k}_\perp)}{2M_N^2 M_h} h_{1T}^\perp H_1^\perp\right], \quad (6)$$

where a compact notation

$$\mathcal{F}[\omega f D] = \sum_q e_q^2 \int d^2\mathbf{p}_\perp d^2\mathbf{k}_\perp \delta^2(\mathbf{p}_\perp - \mathbf{k}_\perp - \mathbf{P}_{h\perp}/z) \\ \omega(\mathbf{p}_\perp, \mathbf{k}_\perp) f^q(x, \mathbf{p}_\perp^2) D^q(z, z^2 \mathbf{k}_\perp^2) \quad (7)$$

is used. We might as well take the $\sin(\phi_h + \phi_S)$ asymmetry as an example, and the other asymmetries can be analyzed in the same way. The asymmetry for a ${}^3\text{He}$ target can be calculated from

$$A_{{}^3\text{He}}^{\sin(\phi_h + \phi_S)}(x, z, \mathbf{P}_{h\perp}) = \frac{\int dy \frac{2\alpha^2}{sxy^2} (1-y) \mathcal{F}\left[-\frac{\hat{\mathbf{h}} \cdot \mathbf{k}_\perp}{M_h} h_1^{{}^3\text{He}} H_1^\perp\right]}{\int dy \frac{2\alpha^2}{sxy^2} (1-y + \frac{1}{2}y^2) \mathcal{F}[f_1 {}^3\text{He} D_1]}, \quad (8)$$

while for a proton or a neutron target, this can be written as

$$A_{p(n)}^{\sin(\phi_h + \phi_S)}(x, z, \mathbf{P}_{h\perp}) = \frac{\int dy \frac{2\alpha^2}{sxy^2} (1-y) \mathcal{F}\left[-\frac{\hat{\mathbf{h}} \cdot \mathbf{k}_\perp}{M_h} h_1^{p(n)} H_1^\perp\right]}{\int dy \frac{2\alpha^2}{sxy^2} (1-y + \frac{1}{2}y^2) \mathcal{F}[f_1^{p(n)} D_1]}. \quad (9)$$

Now we rewrite the expression for the dilution factors in a more complete form

$$f_{p(n)}(x, z, \mathbf{P}_{h\perp}) = \frac{\int dy \frac{2\alpha^2}{sxy^2} (1-y + \frac{1}{2}y^2) \mathcal{F}[f_1^{p(n)} D_1]}{\int dy \frac{2\alpha^2}{sxy^2} (1-y + \frac{1}{2}y^2) \mathcal{F}[(2f_1^p + f_1^n) D_1]}. \quad (10)$$

Substituting Eqs. (9) and (10) into Eq. (1), we have

$$A_{{}^3\text{He}}^{\sin(\phi_h + \phi_S)}(x, z, \mathbf{P}_{h\perp}) = \frac{\int dy \frac{2\alpha^2}{sxy^2} (1-y) \mathcal{F}\left[-\frac{\hat{\mathbf{h}} \cdot \mathbf{k}_\perp}{M_h} (2p_p h_1^p + p_n h_1^n) H_1^\perp\right]}{\int dy \frac{2\alpha^2}{sxy^2} (1-y + \frac{1}{2}y^2) \mathcal{F}[(2f_1^p + f_1^n) D_1]}. \quad (11)$$

TABLE I: kinematics

6 GeV	12 GeV
$1.3\text{GeV}^2 < Q^2 < 3.1\text{GeV}^2$	$Q^2 > 1\text{GeV}^2$
$5.4\text{GeV}^2 < W^2 < 9.3\text{GeV}^2$	$W^2 > 2.3\text{GeV}^2$
$0.13 < x < 0.4$	$0.05 < x < 0.65$
$0.68 < y < 0.86$	$0.34 < y < 0.9$
$0.46 < z < 0.59$	$0.3 < z < 0.7$

Comparing Eq. (11) with Eq. (8), we find that it implies the equalities

$$\begin{aligned} h_1^{^3\text{He}} &= 2p_p h_1^p + p_n h_1^n, \\ f_1^{^3\text{He}} &= \sum_{N=p,n} f_1^N = 2f_1^p + f_1^n, \end{aligned} \quad (12)$$

which can be understood as the impulse approximation to treat scattering on both unpolarized and polarized ${}^3\text{He}$ as an incoherent sum of scattering on free nucleons. The first line of the above equalities shows that the transversity distribution of ${}^3\text{He}$ is a linear combination of the free nucleon transversity with the effective polarizations as their coefficients. We take it as a reasonable assumption that can be generalized to other polarized distributions such as pretzelosity. The second line of the equalities implies that for the unpolarized distribution, ${}^3\text{He}$ can be treated as a sum of free nucleons. Strictly speaking, this is not accurate because the nuclear effects such as the EMC effect [12] are neglected. However, given that ${}^3\text{He}$ is a few body system, the EMC effect is not significant, and thus we could admit that the second line of the above equalities holds as an approximation. So Eq. (1) is equivalent to Eq. (12). We could also understand it in the following way: we view Eq. (12) as our general assumption, then starting from this equation, we can derive Eq. (1) from the definition of the asymmetries for the proton (neutron) and ${}^3\text{He}$. However, we should not take the dilution factors as fixed constants as in our previous calculation.

In the next calculation, the TMD distributions will be derived from an SU(6) quark-diquark model as we used in our previous papers [13, 14], with the Collins function and the unpolarized fragmentation function parametrized from Anselmino *et al.* [15] and Kretzer *et al.* [16], respectively. The kinematics for the 6 and 12 GeV are in Table I.

First, we will present our numerical results on the dilution factors under 6 and 12 GeV

kinematics in Fig. 1 and Fig. 2, respectively. From the two figures, we find that the dilution factors vary very little with z and $P_{h\perp}$, but vary significantly with x , especially for the π^+ production. In our previous calculation [8], we used the estimation $f_p \approx 0.34$ and $f_n \approx 0.32$ as proposed [9]. We are mainly interested in the x dependence, and could make a comparison with our results. For the π^- production, our results are very close with that in the proposal [9]. However, for the π^+ production, we give a much smaller dilution factor for neutron and a larger one for proton, especially when x increases. This can be easily understood that for the π^+ production, $u \rightarrow \pi^+$ is favored and $d \rightarrow \pi^+$ is unfavored. Since the ${}^3\text{He}$ target can be viewed as a nearly free neutron target, we expect that our updated results of the asymmetries for the π^+ production might be suppressed due to this new treatment with the dilution factors.

Fig. 3 and Fig. 4 shows the $\sin(\phi_h + \phi_S)$ and the $\sin(3\phi_h - \phi_S)$ asymmetries at JLab 6 GeV and 12 GeV, respectively. From the figures, we can see that the asymmetries for the ${}^3\text{He}$ target are rather small. This can be accounted for by the suppression of the dilution factors. We deal with the dilution factors in two approaches, one is to use the constant value estimation, i.e. $f_n \approx 0.32$ and $f_p \approx 0.34$ as was supposed to hold around $z \approx 0.5$ [9] and used in our previous calculation [8], and the other is to calculate from the definition in Eq. (10), where the dilution factors vary with x , z and $P_{h\perp}$. We find that the results of the two approaches are very close to each other for the z and $P_{h\perp}$ dependence. However, it has some difference for the x dependence. As we expected, the asymmetry for the π^- production in both approaches are also very close to each other. But for the π^+ production, the asymmetry in the latter approach is indeed smaller than that in the former approach according to our analysis in the above. Such small results might be achieved in JLab with a high precision measurement, so we should be careful about this effect caused by the dilution factors. Also we plot the $\sin(3\phi_h - \phi_S)$ asymmetry in the lower panels in Fig. 3 and Fig. 4. This asymmetry is indeed much smaller than the $\sin(\phi_h + \phi_S)$ asymmetry as we expected in our previous calculation [14]. This implies that it might be more difficult to measure the $\sin(3\phi_h - \phi_S)$ asymmetry related to the pretzelosity, unless a high precision can be achieved. It is suggested [14] that the asymmetry can be amplified by introducing a transverse momentum cut in data analysis.

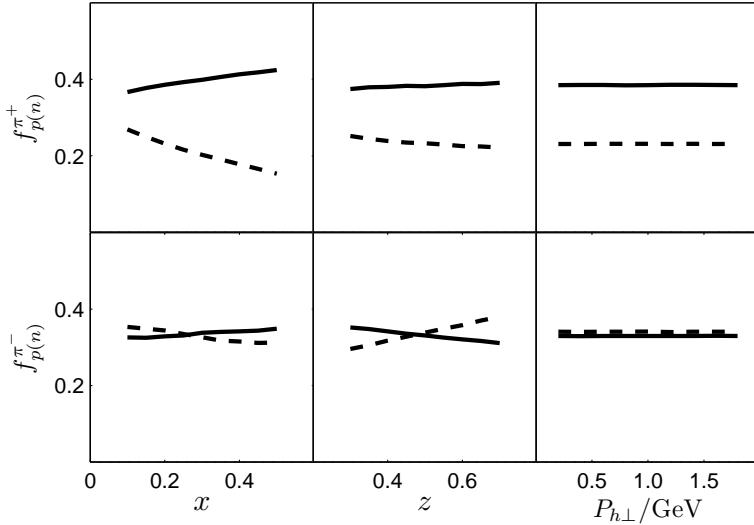


FIG. 1: Dilution factors for π production off a ${}^3\text{He}$ target at JLab 6 GeV kinematics. The solid and dashed curves are the results for the proton and neutron dilution effects, respectively.

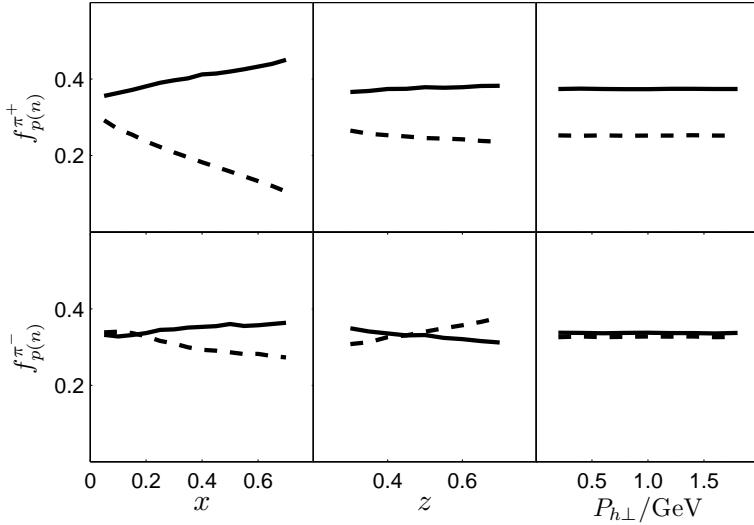


FIG. 2: Similar as Fig. 1, but for the 12 GeV kinematics.

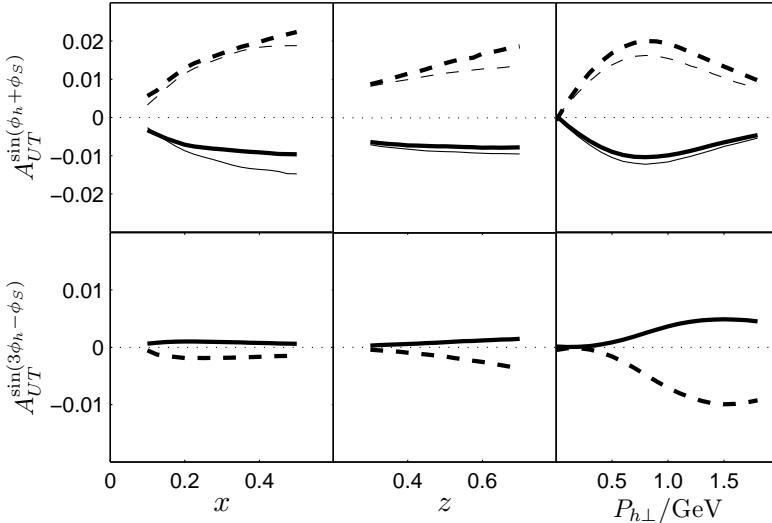


FIG. 3: $\sin(\phi_h + \phi_S)$ and $\sin(3\phi_h - \phi_S)$ asymmetries for π production off a ${}^3\text{He}$ target at JLab 6 GeV kinematics. The solid and dashed curves are the results for the π^+ and π^- production, respectively. The thin curves are the results in which we make the assumption that the dilution factors are constants as we did in our previous paper.

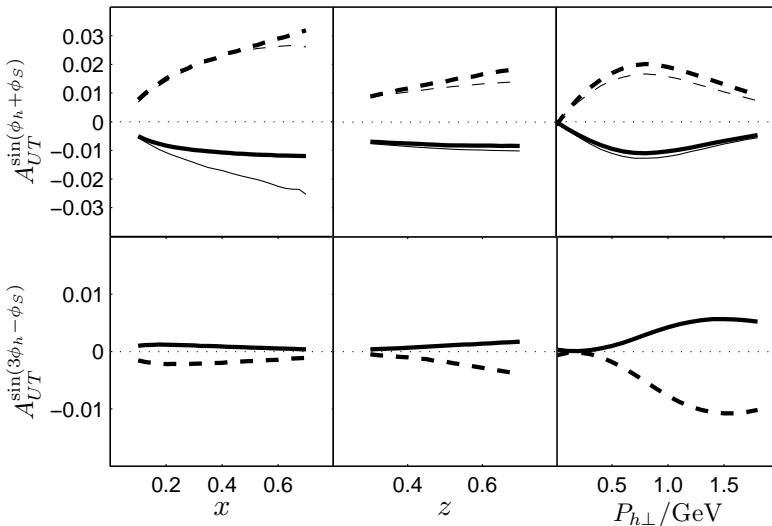


FIG. 4: Similar as Fig. 3, but for the 12 GeV kinematics.

III. CONCLUSION

Eq. (1) was supposed as a useful formula to extract the neutron information in the experiment. Using mean values for the dilution factors in the calculation, we find that such treatment might be approximately valid for the z and $P_{h\perp}$ dependence, and even for the x dependence in the π^- production, but cannot be applied for the x dependence in the π^+ production. In this note, we performed the calculation starting from the definition (10), which is demonstrated to be equivalent to the relation (12), and find that this approach may cause smaller results of asymmetries than our previous calculation [8] for the π^+ production process. So we suggest that in extracting the neutron information, it is not adequate to use the mean values for the dilution factors. Unreliable treatment of dilution factors may bring unreliable information of the neutron quantities extracted from ${}^3\text{He}$ data. Therefore it is important to find reliable treatment of dilution factors in extracting the neutron asymmetries from ${}^3\text{He}$ experiments. Due to the suppression from the dilution factors, all the ${}^3\text{He}$ asymmetries are predicted to be very small, so high precision measurements are demanded.

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